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Hidden Markov Modeling for Radar Electronic Warfare

Pierre Lavoie

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TECHNICAL MEMORANDUM
DREO TM 2001-127
October 2001

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Defence Research Establishment Ottawa

Technical Memorandum

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Abstract

The growing proliferation and complexity of electromagnetic signals encountered in modern environments is greatly complicating the Electronic Support (ES), Electronic Attack (EA) and Electronic Intelligence (ELINT) functions. In conventional ES, EA and ELINT modeling, radar signals are expected to be stable or to correspond to radar modes. The concept examined in this report is the recognition of radar systems using hidden Markov modeling. The model is called hidden because the input symbols and state transitions of the radar system cannot be observed. What is observed is the output of the finite state automaton. This concept provides flexibility for modeling the radar system and preserves a maximum amount of information about it. Four electronic warfare problems are examined: (1) the classification problem: given an observation signal and multiple competing radar models, how do we choose the model which best matches the observation signal? (2) the decoding problem: given the observation sequence and the radar model, how do we choose a state sequence which in some way best explains the observations? (3) the prediction problem: given a partial observation sequence and the radar model, how do we predict the next observation symbol, and compute the associated probability? and (4) the training problem: given only the observation sequence, how do we adjust (or train) the radar model so that it best describes how the observations came about? Known solutions for the first three problems are reviewed. The fourth problem is more difficult. Solving it would grant ES, EA and ELINT systems the capability to automatically reverse-engineer radar systems from intercepted radar waveforms.

Résumé

La prolifération et complexité grandissante des signaux électromagnétiques rencontrés dans les environnements modernes compliquent considérablement les systèmes de soutien électronique, d'attaque électronique, et de surveillance électronique. Dans les systèmes conventionnels, les signaux radars sont présumés être stables ou correspondre à des modes radar. Le concept examiné dans ce rapport est la reconnaissance des systèmes radar avec la modélisation markovienne cachée. Le modèle est dit caché parce que les symboles d'entrée et les transitions d'états du système radar ne peuvent pas être observés. Ce qui est observé est la sortie de l'automate à états finis. Ce concept procure une grande souplesse dans la modélisation du système radar et préserve une quantité maximale d'information à son sujet. Quatre problèmes de la guerre électronique sont examinés: (1) le problème de la classification: étant donné l'observation d'un signal et plusieurs modèles radar en compéti-

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tion, comment choisit-on le modèle qui se marie le mieux avec le signal observé? (2) le problème du décodage: étant donné une suite d'observations et un modèle radar, comment choisit-on une séquence d'états qui d'une certaine façon explique le mieux possible la suite d'observations? (3) le problème de la prédiction: étant donné une suite d'observations et un modèle radar, comment peut-on prédire les observations futures, et les probabilités correspondantes? et (4) le problème de l'apprentissage: étant donné seulement une suite d'observations, comment peut-on ajuster ou apprendre le modèle radar qui explique le mieux possible la genèse des observations? Des solutions connues aux trois premiers problèmes sont revues. Le quatrième problème est plus difficile. Le résoudre donnerait aux systèmes de guerre électronique la capacité d'effectuer automatiquement l'ingénierie inverse des systèmes radar à partir des signaux interceptés.

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Executive summary

This is the first report of the Technology Investment Fund (TIF) Project Hac14 entitled Adaptive Learning Techniques for Future Radar and Communications ESM.

The growing proliferation and complexity of electromagnetic signals encountered in modern environments is greatly complicating the Electronic Support (ES), Electronic Attack (EA) and Electronic Intelligence (ELINT) functions. Conventional ES, EA and ELINT are based on assumptions that are no longer valid: that signals are stable, and correspond to emitter modes. Modes date back to the early days of radar when an operator would change the signal by manually switching to another electrical circuit. Nowadays, radar signals are automatically adjusted by computer software to maximize radar performance. The signals are no longer stable, and modes have lost their meaning. Therefore new, sophisticated ES, EA and ELINT signal processing algorithms are needed to exploit the electromagnetic spectrum of today and tomorrow.

In this report, a new concept is examined that departs from conventional approaches by requiring neither that radar signals be stable nor that they correspond to modes. The concept is to idealize each radar system as a finite state automaton. This provides flexibility for modeling the radar system, and preserves a maximum amount of information about it.

The radar finite state automaton can be modeled using a hidden Markov model. The model is called hidden because the input symbols and state transitions of the radar system cannot be observed by the electronic warfare system. What can be observed is the output of the finite state automaton. Hidden Markov models have been used extensively, and very successfully, for speech recognition.

From a user's standpoint, this new concept appears promising to achieve:

1. Complete representation of radar systems in threat libraries,
2. Fast and accurate classification of radar emitters,
3. Warning of events such as detection by the radar system and missile launch,
4. Improved electronic attack using non-causal transmission of jamming pulses,
5. Reverse-engineering of radar systems, and
6. Automatic training of threat libraries from intercepted radar waveforms.

This concept is compatible with conventional receiver front-ends, and could eventually be inserted into a wide range of legacy ES, EA and ELINT systems. The only hardware requirement is the availability of a fast processor with sufficient memory.

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Pierre Lavoie; 2001; Hidden Markov Modeling for Radar Electronic Warfare; DREO TM 2001-127; Defence Research Establishment Ottawa.

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Sommaire

Ceci est le premier rapport du projet numéro 11ac14 intitulé Techniques d'apprentissage adaptatif pour les mesures de soutien électronique radar et communications de l'avenir.

La prolifération et complexité grandissante des signaux électromagnétiques rencontrés dans les environnements modernes compliquent considérablement les fonctions de soutien électronique, d'attaque électronique, et de surveillance électronique. Ces fonctions sont présentemment basées sur des hypothèses qui ne sont plus valides, que les signaux sont stables, et correspondent à des modes radar. Les modes datent de l'origine du radar lorsque l'opérateur devait changer le signal transmis manuellement par commutation d'un circuit électrique. De nos jours, les signaux radar sont ajustés automatiquement par logiciel d'ordinateur pour maximiser la performance radar. Les signaux ne sont plus stables, et les modes ont perdu leur signification. Conséquement de nouveaux algorithmes sophistiqués de support électronique, d'attaque électronique et de renseignement électronique sont requis pour exploiter le spectre électromagnétique d'aujourd'hui et de demain.

Dans ce rapport, un nouveau concept est examiné qui se distingue des approches conventionnelles en ne requérant ni que les signaux radar soit stables, ni qu'ils correspondent à des modes. Le concept est d'idéaliser chaque système radar comme un automate à états finis. Ceci procure une grande souplesse dans la modélisation du système radar, et préserve une quantité maximale d'information à son sujet.

L'automate à états finis radar peut être modélisé en utilisant un modèle de Markov caché. Le modèle est dit caché parce que les symboles d'entrée et les transitions entre les états du système radar ne peuvent être observés par le système de guerre électronique Ce qui peut être observé est la sortie de l'automate à états finis. Les modèles de Markov cachés ont été utilisés fréquemment, et avec beaucoup de succès, pour la reconnaissance de la parole.

Du point de vue de l'utilisateur, ce nouveau concept paraît prometteur pour:

1. La représentation complete des systèmes radar dans les librairies de menaces,
2. La classification rapide et correcte des émetteurs radar,
3. L'alerte d'évènements tels que la détection par le système radar et le lancement de missiles,
4. L'amélioration de l'attaque électronique par la transmission non-causale d'impulsions de brouillage,
5. L'ingénierie inverse des systèmes radar, et

6 L'apprentissage automatique des librairies de menaces à partir des formes d'ondes radar interceptées.

Ce concept est compatible avec les récepteurs conventionnels et pourrait éventuellement être inséré dans un large éventail de systèmes de soutien électronique, d'attaque électronique et de renseignement électronique. Le seul prérequis matériel est la disponibilité d'un processeur rapide avec suffisamment de mémoire.

Pierre Lavoie; 2001; Modélisation markovienne cachée pour la guerre électronique radar; DREO TM 2001-127; Centre de recherches pour la défense, Ottawa.

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1 Background of the Research

In current electronic warfare (EW) systems, radar signals are recognized using histograms of the pulses in some parametric space (for example, frequency, pulse width and angle of arrival), and temporal periodicities in the pulse train.

Andersen and Murphy, in U.S. Patent No. 5,583,505 [1], describe a radar pulse detection and classification system that receives times-of-arrival of pulses from simultaneous emitters, deinterleaves them into bands of pulse repetition intervals, and determines the pulse periodicities by autocorrelation.

Caschera, in U.S. Patent No. 5,063,385 [11], describes a memory system for histogramming the pulse descriptor word output of a radar warning receiver for quickly determining the number and types of emitters in the system's environment.

Maier, in U.S. Patent No. 4,918,455 [25], describes the deinterleaving of sequential signal pulses from unknown sources by clustering similar pulses into groups, and the use of such groups to form hypothetical pulse train models.

Dunne and Gentry, in U.S. Patent No. H513 [12], describe a tracking apparatus that uses multi-processor modules for predicting in read time the parametric behavior of radar signals to be jammed.

A drawback of histogramming on parameters available on individual pulses is that the temporal relationship of the pulses is lost in the histogram. A limitation of periodic temporal analysis is that it assumes that the radar system is a periodic source of pulses. This holds true only for simple radar systems or for short periods of time.

Taken separately or together, these approaches are ill-suited to exploit the fact that the vast majority of radar systems are dynamic. Indeed, in response to various events, radar systems can change their transmitted signal Let us illustrate this with some examples.

- The operator turns a control knob to select a different range display,
- The radar system detects a target and switches from a search to a tracking signal,
- The radar software switches between a number of periodic signal patterns to reduce blind ranges and speeds, and
- The launch of a missile triggers the transmission by the radar system of a guidance signal.

This research is not about the events themselves so much as about acknowledgment of their existence, and exploitation of the opportunities they create for EW.

For example, the radar system may change its pulse repetition interval (PRI) from one stable value to another. Given sufficient observations, one could determine in what sequence PRI changes can and cannot occur, and what are the time intervals separating the difference PRIs. This information may be very useful to recognize one radar system among others that use the same stable PRI values but in a different order or with different transition intervals.

The radar system may transmit a sequence of pulses spaced by varying time intervals. Given sufficient observations, we could determine in what order the pulse to pulse intervals can and cannot occur. For instance, the sequence of intervals could be periodic, exhibit partial freedom, or be entirely pseudo-random.

In addition to changing the pulse to pulse interval, many radar systems can transmit several different pulses. The choice of pulse combined with the choice of interval by the radar system can disclose further information. For example, two radar systems may have the same set of pulses and intervals, but still be recognizable from each order it the order in which they transmit and space their pulses differ.

Given the dynamic nature of radar systems, it is only natural to exploit this dimension in EW systems. The approach examined here is to treat radar systems as finite state automata, and to try to recognize them using hidden Markov modeling.

Radar recognition using hidden Markov modeling is a recent idea. Upon circulation of a draft copy of this report, the U.K. Defence and Science Technology Laboratory brought to the attention of the author their related research programme which ran from 1994 to 1996. Nayar and Smithers, in [26], describe the use of hidden Markov models to recognize corrupted sequences of pulse descriptor words from a deinterleaver. To the best of the author's knowledge, this is the first reference to hidden Markov modeling of the radar itself. Although they obtained mixed results, Nayar and Smithers therefore pioneered in the development of this concept.

2 Radar Systems as Finite State Automata

From [15], "a finite automaton consists of a finite set of states and a set of transitions from state to state that occur on input symbols chosen from an alphabet. For each input symbol, there is exactly one transition out of each state (possibly back to the state itself)."

The aspects of radar systems that are of interest to us can be captured in finite state automata. Naturally, the input symbols and state transitions of the radar system are

likely to be unobservable by the EW system. What can be observed by the EW system is the output of the finite state automaton. This output is chosen from an alphabet containing, for example, pulses and time intervals. The output may be associated with the state (called a Moore machine) or with the transition (called a Mealy machine). Whether the radar system corresponds to a Moore or a Mealy machine is irrelevant here, because it has been shown that for any Moore machine, there is an equivalent Mealy machine, and vice versa [15].

The state diagram of a Moore machine is shown in Figure 1. A state diagram is a

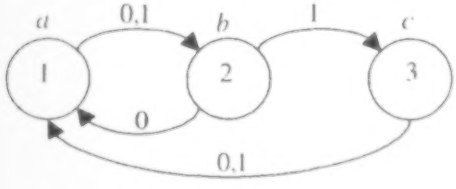

Figure 1: *Transition diagram of a Moore machine*

directed graph where the vertices of the graph correspond to the states of the finite state automaton. If there is a transition from state i to state j on input x, then there is an arc labeled x from state i to state j in the transition diagram.

The finite state automaton of Figure 1 has three states, accepts a binary input from alphabet $\{0, 1\}$, and produces a ternary output from alphabet $\{a, b, c\}$. Outputs are shown above the states. If the automaton is in state 1 and reads an input, then it necessarily enters state 2 because the corresponding arc is labeled with both 0 and 1. There are two arcs leaving state 2, one labeled 0 going to state 1, and one labeled 1 going to state 3. The next state therefore depends on the input symbol read by the automaton: it will be state 1 if the automaton reads 0 and state 3 if the automaton reads 1. If the automaton is in state 3 and reads an input, then it goes to state 1. Starting in state 1 and reading an input sequence of consecutive 0s, the automaton produces the output sequence $ababab\cdots$. Starting in the same state and reading an input sequence of consecutive 1s, it produces $abcabcabc\cdots$.

The finite state automaton representation of the radar system is a convenient way to analyze its dynamic behavior for three reasons.

First, the dynamic behavior of the radar system is readily apparent upon examination of the state transitions. Every time the finite state automaton is in a state from which transition is possible to two or more states, the next state is selected based on the input symbol. The input symbol can be randomly chosen, controlled by a tracking algorithm, controlled by the operator, and so on. This reflects the fact that the radar system is driven by events. If the finite state automaton contains no state

from which transition is possible to two or more states, then the corresponding radar system is not dynamic, and it necessarily produces a periodic signal.

Second, the finite state automaton representation is flexible since the output alphabet may contain pulses, intervals, pulses together with intervals, bursts of consecutive pulses, portions of pulses, portions of continuous wave (CW) signals, and so on. Moreover, symbols of the output alphabet need not have the same duration.

Third, as we are about to see, idealizing the radar system as a finite state automaton allows some powerful analysis using hidden Markov models.

3 Hidden Markov Models

3.1 Definition

Hidden Markov models (HMMs) are described in [27, 28, 18]. They have been used extensively, and very successfully, for speech recognition.

Jelinek, in [17], describes statistical methods of automatic recognition of continuous speech that concern the modeling of a speaker and of an acoustic processor, extraction of the models' statistical parameters, and hypothesis search procedures and likelihood computations of linguistic decoding.

Levinson, Rabiner and Sondhi, in U.S. Patent No. RE33,597 [22], describe a speech recognizer that includes storing a set of signals, each representative of a prescribed acoustic feature and storing a template for each spoken reference word which comprises signals representative of a first state, a last state and preselected number of intermediate states of a constrained hidden Markov model.

Doddington and Pawate, in U.S. Patent No. 4,977,598 [13], describe an efficient pruning algorithm which reduces computer processing unit loading during speech recognition by means of a hidden Markov model.

Bahl, Brown, deSouze and Mercer, in U.S. Patent No. 4,827,521 [3], describe the training of Markov models, preferably in a speech recognition environment, in a manner directed toward maximizing word decoding accuracy by maximizing the difference between the probability of the correct script of uttered words given the label outputs and the probability of any other (incorrect) script, rather than maximizing the probability of the labels given the script.

Hidden Markov models have also been used for pulse train deinterleaving. Logothetis and Krishnamurthy, in [24], describe the deinterleaving of pulse trains transmitted by independent sources, by modeling the signal from each source as an in-

dependent stationary auto-regressive process, and then combining a hidden Markov model estimator with a binary time-series estimator to obtain maximum likelihood estimates of the coefficients of the auto-regressive sources.

From [28], an HMM is characterized by the following:

1. N, the number of states in the model. The states are hidden, that is, they are not observable. The individual states are labeled as $\{1, 2, \cdots N\}$, and the state at time t is q_t.

2. M, the number of distinct observation symbols per state, that is, the discrete alphabet size. The observation symbols correspond to the physical output of the system being modeled. The individual symbols are denoted as $V = \{v_1, v_2, \cdots, v_M\}$

3. A state transition probability distribution $A = \{a_{ij}\}$ where

$$a_{ij} = P[q_{t+1} = j | q_t = i], \quad 1 \leq i, j \leq N \tag{1}$$

   For the special case where any state can reach any other state in a single step, we have $a_{ij} > 0$ for all i, j. In other cases, we would have $a_{ij} = 0$ for one or more (i, j) pairs.

4. The observation symbol probability distribution $B = \{b_j(k)\}$, in which

$$b_j(k) = P[o_t = v_k | q_t = j], \quad 1 \leq k \leq M, \tag{2}$$

   defines the symbol distribution in state j, $j = 1, 2, \cdots, N$, and o_t denotes the observation symbol at time t

5. The initial state distribution $\pi = \{\pi_i\}$ where $\pi_i = P[q_1 = i], 1 \leq i \leq N$.

Given appropriate values of N, M, A, B, and π, the HMM can be used as a generator to give an observation sequence $O = (o_1 o_2 \cdots o_T)$ where each observation o_t is one of the symbols from V, and T is the number of observations in the seuence. For brevity, the compact notation $\lambda = (A, B, \pi)$ is used to denote the complete parameter set of the model.

3.2 Hidden Markov Modeling of Finite State Automata

There are many similarities between finite state automata and HMMs. The states and the outputs of the finite state automaton can be modeled as states and observation symbols in the HMM. The main difference is that while transitions are driven

by input symbols in the finite state automata, they occur statistically in HMMs. For instance, every transition out of every state (possibly back to the state itself) in the finite state automaton is modeled as a non-zero transition probability in the probability distribution A of the HMM. The lack of a transition in the finite state automaton is modeled as a zero transition probability in A. This difference accommodates the fact that input symbols and state transitions are unobservable from an EW standpoint.

To illustrate the difference between finite state automata and HMMs, we return to the Moore machine of Figure 1. Many Markov models could explain sequences output by this finite state automaton. We consider three possibilities.

The model shown in Figure 2 has $N = 3$ states, each state corresponding to one

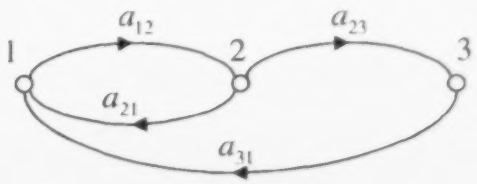

Figure 2: *A possible Markov model which can account for observation sequences produced by the finite state automaton of Figure 1.*

of $M = 3$ observation symbols, $v_1 = a$, $v_2 = b$ and $v_3 = c$, respectively, so that $b_1(1) = b_2(2) = b_3(3) = 1$. The observation alphabet of this model is identical to the output alphabet of the finite state automaton. One possible state transition probability distribution is given by

$$A = \{a_{ij}\} = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0 \end{bmatrix}$$

where transition from state 1 is always to state 2, transition from state 3 is always to state 1, and transitions from state 2 to states 1 and 3 have been chosen to be equiprobable.

The model shown in Figure 3 has only $N = 2$ states, corresponding to $M = 2$ observation symbols, $v_1 = ab$ and $v_2 = abc$, respectively. In this model the observation alphabet is different from the output alphabet of the finite state automaton. One possible state transition probability distribution A is given by

$$A = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

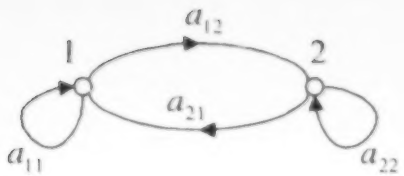

Figure 3: *Another possible Markov model which can account for observation sequences produced by the finite state automaton of Figure 1.*

where all transitions have been chosen to be equiprobable.

Before introducing the third model, we examine modeling for discrete time and uniform symbol duration.

3.3 Discrete Time and Uniform Symbol Duration

In practice, most radar systems contain a clock, which clock is used to produce pulse to pulse intervals that are integer multiples of the clock period. Generally, the radar clock period, or a multiple thereof, can be estimated by the EW System. We denote this estimated period by τ, and assume for simplicity that it is exact.

Having estimated the radar clock period, the EW system can transform the observation signal into a sequence of observation symbols, $O = (o_1 o_2 \cdots o_T)$, where the duration of each symbol o_t is equal to some integer multiple of the time period τ. This implies proper synchronization, and an observation alphabet V where symbol durations are all multiples of τ.

Note that for radar systems that do not use a clock, either τ can be chosen small, or a continuous-time HMM can be used.

To illustrate discrete-time modeling, we define for the finite state automaton of Figure 1 the discrete-time output alphabet

symbol	duration	description
a	1τ	a pulse
b	2τ	symbol a followed by one empty interval
c	3τ	symbol a followed by two empty intervals.

Denoting by 1 an interval containing a pulse, and by 0 an empty interval, the output alphabet can be rewritten as $a = 1$, $b = 10$ and $c = 100$.

We are now ready to introduce the third model, which can account for sequences produced by the discrete-time finite state automaton. This model is essentially the Markov model of Figure 2 modified to accept observation symbols of uniform duration τ.

Define $M = 2$ new observation symbols, $v_1 = 1$ and $v_2 = 0$, having the same duration τ, and corresponding to an interval containing a pulse, and an empty interval, respectively. We can transform each state of the original HMM into a sequence of one or more states in the new HMM. The size of each sequence is set equal to the duration (in clock periods) of the symbol of the original state. The sequence itself is a left-right model with an entry state, intermediate states, an exit state, and a single transition with probability 1 between each state and the next, beginning with the entry state, and ending with the exit state. We can then take each transition of the original HMM, and replicate it between the corresponding sequences in the new HMM, and more precisely from the exit state of the first sequence to the entry state of the second sequence.

Starting from Figure 2, state 1 with probability distribution $b_1(1) = 1$, symbol $v_1 = a = 1$ and duration τ, is transformed into a sequence of one state, denoted state $1'$. State 2 with $b_2(2) = 1$, symbol $v_2 = b = 10$ and maximum duration 2τ, is transformed into a sequence of two states $2'$ and $3'$. State 3 with $b_3(3) = 1$, $v_3 = c = 100$ and maximum duration 3τ, is transformed into a sequence of three states $4'$, $5'$ and $6'$. The transitions in the sequences are $a_{2'3'} = 1$, $a_{4'5'} = 1$ and $a_{5'6'} = 1$. The original transitions a_{12}, a_{21}, a_{23}, and a_{31} become $a_{1'2'}$, $a_{3'1'}$, $a_{4'1'}$, and $a_{6'1'}$, respectively. Lastly, the symbols corresponding to the new states $1'$, $2'$, $\ldots$, $6'$ are set equal to 1, 1, 0, 1, 0 and 0. To summarize, the state transition probability distribution is

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and the states and possible transitions of the new HMM are shown in Figure 4.

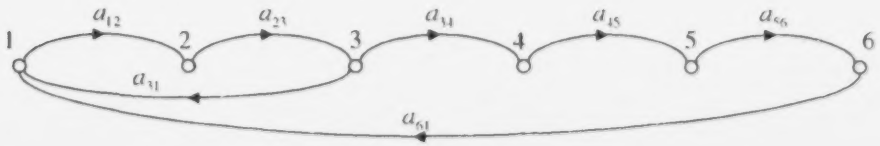

Figure 4: *Markov model of Figure 2 modified to accept observation symbols of uniform duration*

If the original HMM contains one (or more) states where symbols of different durations have positive probabilities, then the transformation into a uniform discrete-time HMM is still possible. However the procedure is more involved and beyond the scope of this document.

3.4 Observation Errors

In the first two Markov models, we associated a single possible observation symbol to each state of the HMM. However in reality errors do occur during the observation of radar systems. One way to account for these errors in the model is by means of the observation symbol probability distribution B.

For example, it may occur that a pulse is observed when the radar output was an empty interval. Borrowing from radar nomenclature, we call this event a false alarm and denote the false-alarm probability as P_F. Conversely, it may occur that nothing is observed when the radar output was a pulse. We call this a miss, and denote the miss probability as P_M. Formally, we have

$$P_F = P[o_t = 1 | \text{output 0 at } t] \tag{3}$$

$$P_M = P[o_t = 0 | \text{output 1 at } t] \tag{4}$$

Then for the Markov model of Figure 4, the observation symbol probability distribution $B = \{b_j(k)\}$, $b_j(k) = P[o_t = v_k | q_t = j]$, $j = 1, 2, \cdots, 6$, is given by

j	v_k	$b_j(k)$
1	1	$1 - P_M$
1	0	P_M
2	1	$1 - P_M$
2	0	P_M
3	1	P_F
3	0	$1 - P_F$
4	1	$1 - P_M$
4	0	P_M
5	1	P_F
5	0	$1 - P_F$
6	1	P_F
6	0	$1 - P_F$

This last model is a uniform discrete-time HMM that can account for spurious and missing pulses.

3.5 Trellis Representation

Given the states and transitions of a uniform discrete-time HMM, we can construct a graph called the trellis diagram as follows. First, we replicate T copies of the N states. The states of the tth copy correspond to the HMM at time t. An arc connects a state A_i of the tth copy with a state A_j of the $(t+1)$th copy if the corresponding transition probability a_{ij} is positive.

For example, given the Markov model of Figure 4, we can construct the trellis shown in Figure 5.

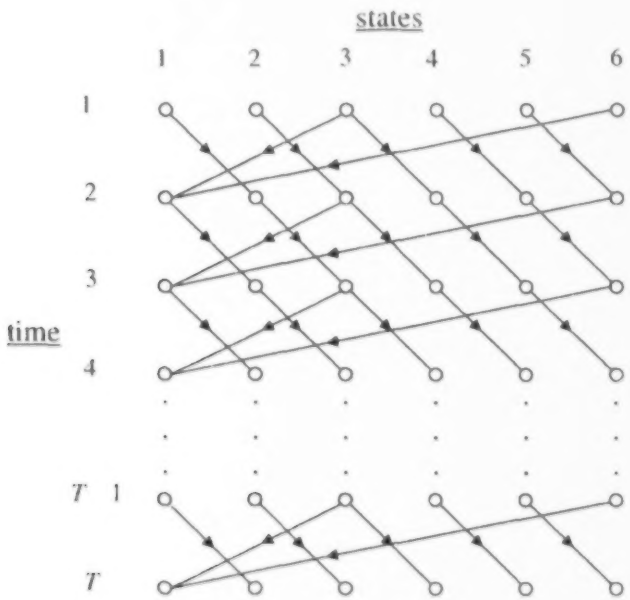

Figure 5: *Trellis derived from the Markov model of Figure 4.*

The main advantage of the trellis diagram is that it leads to fast algorithmic solutions to many problems of interest.

3.6 Multivariate Observations

We have so far limited our discussion to the observation of discrete symbols chosen from a finite alphabet. It is important to note that all the concepts presented in this document can be generalized to continuously distributed multivariate observations. It suffices to replace the observation symbol probability distribution by a joint probability density function for which a HMM training procedure has been formulated.

Compliant joint density functions include elliptically symmetric density functions like multivariate Gaussians [23] and multivariate mixtures like Gaussian sums [19]. Since the latter can approximate any finite continuous multivariate density function, it is not surprising to see HMMs applied to a wide range of problems. In EW, generalization to multivariate observations would allow the use of parameters like pulse width and frequency. Albeit important in practice, the generalization is not, however, particularly edifying in the context of this conceptual document. Therefore, for simplicity we shall consider only discrete observations.

4 Four Electronic Warfare Problems

For brevity, we call a hidden Markov model corresponding to a radar system the *radar model*. We assume that the EW system contains L competing radar models. The individual models are denoted as $\lambda^{(1)}, \lambda^{(2)} \cdots \lambda^{(L)}$. The models have different time periods, which are denoted as $\tau^{(1)}, \tau^{(2)}, \ldots, \tau^{(L)}$. Hence, for a given observation signal, there are several observation sequences, which are denoted as $O^{(1)}, O^{(2)}, \cdots O^{(L)}$, owing to the different time periods.

In this research, four electronic warfare problems are examined:

Problem 1 (Classification): Given an observation signal and L radar models $\lambda^{(1)}$, $\lambda^{(2)}, \cdots, \lambda^{(L)}$, how do we choose the model which best matches the observation signal?

Problem 2 (Decoding): Given the observation sequence O and the radar model λ, how do we choose a state sequence $Q = (q_1 q_2 \cdots q_t)$ which in some way best explains the observations?

Problem 3 (Prediction): Given the partial observation sequence $(o_1 o_2 \cdots o_t)$ and the radar model λ, how do we predict the next observation symbol $o_{t+1} = v_k$, and compute the associated probability $P(o_{t+1} = v_k | o_1 o_2 \cdots o_t \ \lambda)$?

Problem 4 (Training): Given only the observation sequence O, how do we adjust (or train) the radar model λ so that it best describes how the observations came about?

The EW motivation behind each problem and some known solutions are now examined.

4.1 The Classification Problem

The classification problem is "given an observation signal and L radar models $\lambda^{(1)}$, $\lambda^{(2)}$, $\cdots$, $\lambda^{(L)}$, how do we choose the model which best matches the observation signal?"

The classification problem is critical for Electronic Support (ES) and Electronic Attack (EA). Current approaches that use histograms of the pulses in some parametric space and temporal periodicities in the pulse train, extract parameters and match them to competing entries in a library. When the extracted parameters fall into an overlap area of two or more entries of the library, the emitter cannot be uniquely identified, resulting in an ambiguity. EW systems are designed to communicate multiple candidate identifications to the operator. Sometimes a friendly emitter can lead to multiple candidate identifications that include a threat. This triggers an unnecessary alert, jeopardizing safety and increasing the risk of fratricide. Since the HMM can preserve more information about the radar system than current approaches, it has the potential to enhance classification.

Since the L competing radar models have different time periods, the EW system must transform the observation signal into not just one, but L sequences of observation symbols $O^{(1)}, O^{(2)}, \cdots O^{(L)}$, where the duration of each symbol $o_t^{(l)}$ of the sequence is equal to some integer multiple of the lth time period $\tau^{(l)}$, $1 \leq l \leq L$.

Formally the classification problem becomes, given $O^{(1)}, O^{(2)}, \cdots, O^{(L)}$ and $\lambda^{(1)}$, $\lambda^{(2)}, \cdots, \lambda^{(L)}$, how do we choose which of the L radar models is the most probable?

From Bayesian decision theory, one of the most useful decision is to maximize the *a posteriori* probability. The maximum a posterior (MAP) decision is to choose

$$\lambda_{\text{MAP}} = \arg\max_{1 \leq l \leq L} P[\lambda^{(l)}|O^{(l)}] \tag{5}$$

Using Bayes rule, we have

$$P[\lambda|O] = \frac{P[O|\lambda]\, P[\lambda]}{P[O]} \tag{6}$$

Since $P[O]$ is not a function of λ, the MAP decision may also be defined as

$$\lambda_{\text{MAP}} = \arg\max_{1 \leq l \leq L} P[O^{(l)}|\lambda^{(l)}]\, P[\lambda^{(l)}] \tag{7}$$

Albeit powerful, the MAP decision requires prior knowledge of the probabilities $P[\lambda^{(l)}]$, $1 \leq l \leq L$. Such knowledge is not always available.

When the probabilities $P[\lambda^{(l)}]$ are not known *a priori*, we might assume that the competing radar models are all equiprobable. Then we can use the maximum likelihood (ML) decision defined as

$$\lambda_{\mathrm{ML}} = \arg\max_{1 \le l \le L} P[O^{(l)}|\lambda^{(l)}] \tag{8}$$

Whether it is for MAP or ML decision, we need to compute for each of the L radar models, the probability of the sequence given the model. This problem is called the evaluation problem: Given an observation sequence $O = (o_1 o_2 \cdots o_T)$ and a radar model λ, how do we compute $P[O|\lambda]$, the probability of the sequence, given the radar model?

It is tempting to try a direct approach to solving the evaluation problem. However since computing the probability of O given a state sequence Q takes on the order of T calculations, and there are N^T possible state sequences, a direct computation of $P[O|\lambda]$ requires on the order of TN^T calculations. Because of this non-polynomial complexity, the direct approach is practical only for small HMMs.

Fortunately, there is a well-known solution to the problem, called the *forward-backward procedure* [5, 6]. This solution requires on the order of N^2T calculations.

Consider the forward variable $\alpha_t(i)$ defined as

$$\alpha_t(i) = P[o_1 o_2 \cdots o_t \ q_t = i|\lambda]. \tag{9}$$

that is, the probability of being in state i at time t and observing the partial sequence $(o_1 o_2 \cdots o_t)$, given the model λ. The forward variable at time $t = 1$ is the joint probability of state i and observation o_1.

$$\alpha_1(i) = \pi_i b_i(o_1), \quad 1 \le i \le N \tag{10}$$

At time $t = 2$ the forward variable is

$$\begin{aligned} \alpha_2(j) &= \left[\sum_{i=1}^{N} \pi_i b_i(o_1) a_{ij}\right] b_j(o_2), \\ &= \left[\sum_{i=1}^{N} \alpha_1(i) a_{ij}\right] b_j(o_2), \quad 1 \le j \le N \end{aligned}$$

Proceeding inductively, we obtain for $1 \le t \le T-1$,

$$\alpha_{t+1}(j) = \left[\sum_{i=1}^{N} \alpha_t(i) a_{ij}\right] b_j(o_{t+1}), \quad 1 \le j \le N \tag{11}$$

Finally, the sum of the terminal forward variables provides the solution to the evaluation problem, that is,

$$P[O|\lambda] = \sum_{i=1}^{N} \alpha_T(i) \tag{12}$$

This procedure owes its efficiency to the trellis representation of uniform discrete-time HMMs.

Once we have solved the evaluation problem, we can compute, for each of the L models in the classification problem, the probability of the sequence given the model, and decide between the models using the MAP or the ML criterion.

A special situation may occur when the ES or EA system encounters in the field either an unknown radar system, or an unknown signal produced by a known radar system. When this situation occurs, we do not want the ES or EA system to choose a model of its library for the unknown signal: instead we want some indication that an unknown has been observed. This is the problem of decision with a reject option. Formally, the decision can then have $(L+1)$ values: $1, 2, \cdots, L$, and "reject." This problem has been studied in the literature

4.2 The Decoding Problem

The decoding problem is "given the observation sequence O and the radar model λ, how do we choose a state sequence $\hat{Q} = (\hat{q}_1 \hat{q}_2 \cdots \hat{q}_T)$ which in some way best explains the observations?"

We call this problem decoding after the similar communication problem which consists in deciding, given a noisy sequence of encoded symbols and the error correction encoder model, what is the sequence of message input bits.

What is particularly useful for ES and EA is not so much the complete state sequence Q, as the recognition of specific state transitions that provide situational information. Let us illustrate this with two examples. When a missile is launched, the associated radar system often changes its signal to guide the missile. In order for the radar signal to change, some state transition must occur in the finite state automaton of the radar. By watching the state transition that corresponds to a missile launch, the ES or EA system could possibly recognize the event and warn that a missile is airborne. Next, when a radar senses a new target, it often changes its signal to estimate the position and velocity of the target. Again in order for the radar signal to change, some state transition must occur in the radar finite state automaton. By recognizing this state transition, the ES or EA system could possibly warn that a target is being acquired.

Assuming that the radar system has already been classified, our motivation behind the decoding problem is to recognize changes in the radar signal, and ultimately to infer what events are happening. If the HMM contains the state transitions that correspond to critical events, then it has the potential to enhance situation awareness.

The solution to the decoding problem depends on our choice of an optimality criterion to determine which is the best sequence. Here we adopt a commonly used criterion which is to maximize $P[Q|O, \lambda]$. This criterion accounts for the probability of occurrence of sequences of states. Therefore it yields a sequence for which all transitions have positive probability, that is, a valid sequence.

The solution to the decoding problem is the *dynamic programming* algorithm [9] (for a recent treatment, see [10]), also known in the communication world as the *Viterbi* algorithm [30]. Like the forward-backward procedure, this solution requires on the order of N^2T calculations. This algorithm is well known. The reader is referred to [28] for a description of the algorithm in the context of HMMs. Bahl, Jelinek and Mercer, in [2], present shortcuts to reduce the amount of computation and storage. If an exhaustive exploration of the trellis is prohibitive, then suboptimal algorithms like the stack algorithm [31, 16] can be used [18].

4.3 The Prediction Problem

The prediction problem is "given the partial observation sequence $(o_1 o_2 \cdots o_t)$ and the radar model λ, how do we predict the next observation symbol $o_{t+1} = v_k$, and compute the associated probability $P[o_{t+1} = v_k | o_1 o_2 \cdots o_t, \lambda]$?"

This problem is of particular interest for EA using deceptive electronic countermeasures techniques. For example, the range gate pull-off technique involves the transmission of false echo pulses just before or after the arrival of the actual radar pulses to create a false target [29]. Repeater jamming is straightforward. However, to simulate an inbound target, the jammer must anticipate the reception of the radar pulse and transmit the false echo pulse before it. For the technique to work, the time of arrival of the radar pulse must be predicted reliably and accurately, something the HMM can help us do.

This problem is closely related to the evaluation problem, and similarly solved. Using the forward variable, the probability that the next symbol be v_k is

$$P[o_{t+1} = v_k | o_1 o_2 \cdots o_t, \lambda] = \frac{P[o_{t+1} = v_k, o_1 o_2 \cdots o_t | \lambda]}{P[o_1 o_2 \cdots o_t | \lambda]} \tag{13}$$

$$= \frac{\sum_{j=1}^{N} \left[\sum_{i=1}^{N} \alpha_t(i) a_{ij} \right] b_j(k)}{\sum_{i=1}^{N} \alpha_t(i)} \tag{14}$$

Therefore the most probable symbol v_k to occur at time $(t+1)$ is

$$\hat{o}_{t+1} = \arg\max_{1 \le k \le M} \left[\sum_{j=1}^{N} \left[\sum_{i=1}^{N} \alpha_t(i) a_{ij} \right] b_j(k) \right] \tag{15}$$

Similarly, we can use the HMM to predict the most probable symbol to occur at time $(t+2)$, $(t+3)$, and so on. We can also use the HMM to estimate the time at which a given symbol (for example, a pulse) is most likely to be observed next.

4.4 The Training Problem

The training problem is "given only the observation sequence O, how do we adjust (or train) the radar model so that it best describes how the observations came about?"

This is a very fundamental problem. Indeed, as we have seen, we need some radar models prior to solving the classification, decoding and prediction problems. Training an HMM can be viewed as a reverse engineering problem: knowing what the radar produces, what is the finite state automaton in the radar? The training problem is of particular interest for electronic intelligence (ELINT), a branch of warfare that is concerned with the interception and analysis of radar signals for the population of databases. The problem is also important for ES and EA, since these tactical systems sometime encounter in the field unknown radar systems, or unknown signals produced by known radar systems.

The training problem is by far the most difficult of the problems examined here. For instance, maximum *a posteriori* estimation of continuous density HMMs is the subject of active research [20, 14, 21].

Ideally we would like to find the maximum *a posteriori* radar model

$$\lambda_{\mathrm{MAP}} = \arg\max_{\lambda} P[\lambda | O] \tag{16}$$

However, the solution to this equation may not be unique, we typically have only partial knowledge of the probabilities $P[\lambda]$ for all possible λs, and the search space consisting of all possible λs can be very large.

The number of states N being unknown, we may try to give different values to N, then given each N search for the corresponding MAP model, and finally choose the best of the models found.

If we lack prior statistics, we can turn to the maximum likelihood model

$$\lambda_{\mathrm{ML}} = \arg\max_{\lambda} P[O|\lambda] \tag{17}$$

The ML problem is considerably simpler than the MAP problem. Still, there is no known way to analytically solve it.

If we assume knowledge of the number of states N, then we can use an iterative solution to the ML problem, called the *Baum-Welch method* [5, 6, 4, 7, 8]. Like other known solutions, it leads to local maxima of the probability of the sequence being observed from the model, $P[O|\lambda]$. In most problems of interest, the optimization surface is complex and has many local maxima, and therefore the solution found, λ_{BW}, is not necessarily optimum. Before describing the iterative solution, we first define the backward variable as

$$\beta_t(i) = P[o_{t+1}o_{t+2}\cdots o_T|q_t = i, \lambda \tag{18}$$

that is, the probability of observing the partial sequence $(o_{t+1}o_{t+2}\cdots o_T)$, given state i at time t and the model λ. The backward variable at time T is arbitrarily defined as

$$\beta_T(i) = 1 \qquad 1 \le i \le N \tag{19}$$

Proceeding inductively, we obtain for $1 \le t \le T-1$,

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(o_{t+1}) \beta_{t+1}(j), \quad 1 \le i \le N \tag{20}$$

We define the probability of being in state i at time t and state j at time $(t+1)$, given the observation sequence and the model, as

$$\xi_t(i,j) = P[q_t = i, q_{t+1} = j|O, \lambda] \tag{21}$$

From the definitions of the forward and backward variables, we can write $\xi_t(i,j)$ in the form

$$\begin{aligned} \xi_t(i,j) &= \frac{\alpha_t(i) a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)}{P[O|\lambda]} \\ &= \frac{\alpha_t(i) a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)}{\sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_t(i) a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)} \end{aligned} \tag{22}$$

Next, we define one more variable

$$\gamma_t(i) = P[q_t = i|O\ \lambda] \tag{23}$$

as the probability of being in state i at time t, given the observation sequence and the model. From the definitions of the variables $\alpha_t(i)$, $\beta_t(i)$ and $\xi_t(i\ j)$, we can write $\gamma_t(i)$ in the form

$$\begin{aligned}\gamma_t(i) &= \frac{\alpha_t(i)\beta_t(i)}{P[O|\lambda]} \\ &= \frac{\alpha_t(i)\beta_t(i)}{\sum_{i\ 1}^{N}\alpha_t(i)\beta_t(i)} \\ &= \sum_{i\ 1}^{N}\xi_t(i.j).\end{aligned} \tag{24}$$

If we sum $\gamma_t(i)$ over time, we get the expected number of transitions from state i. Similarly, if we sum $\xi_t(i.j)$ over time, we get the expected number of transitions from state i to state j.

Based on the above variables and formulas, the following equations can be used to iteratively estimate π, A and B:

$$\bar{\pi}_i = \gamma_t(i) \tag{25}$$

$$\bar{a}_{ij} = \frac{\sum_{t\ 1}^{T-1}\xi_t(i.j)}{\sum_{t\ 1}^{T-1}\gamma_t(i)} \tag{26}$$

$$\bar{b}_j(k) = \frac{\sum_{t\ \text{such that}\ o_t\ v_k}\gamma_t(j)}{\sum_{t\ 1}^{T}\gamma_t(j)} \tag{27}$$

If we denote the current model as $\lambda = (A, B, \pi)$, we can use the equations (25)–(27) to estimate a new model $\bar{\lambda} = (\bar{A}, \bar{B}, \bar{\pi})$. It has been proven that if $\bar{\lambda} \neq \lambda$, then the probability of O being observed from the model has increased, that is, $P[O|\bar{\lambda}] > P[O|\lambda]$. We may therefore set the current model equal to the new model and use equations (25)–(27) again to re-estimate the model. On the other hand, if $\bar{\lambda} = \lambda$, then some limiting point has been reached and the model $\bar{\lambda} = \lambda_{\text{HW}}$ corresponds to a local maximum of $P[O|\lambda]$.

Again, the optimization surface can be very complex and have many local minima. The model λ_{HW} found by the iterative solution therefore depends to a large extent on the initial condition, that is, the radar model λ defined as current in the first estimation.

5 Conclusion

In conventional EW modeling, radar signals are expected to be stable or to correspond to radar modes. The concept examined in this report is the recognition of radar systems using hidden Markov modeling. The model is called hidden because the input symbols and state transitions of the radar system cannot be observed by the EW system. What is observed is the output of the finite state automaton. Hidden Markov models have been used extensively, and very successfully, for speech recognition.

Four electronic warfare problems have been examined: (1) the classification problem: given an observation signal and multiple competing radar models, how do we choose the model which best matches the observation signal? (2) the decoding problem: given the observation sequence and the radar model, how do we choose a state sequence which in some way best explains the observations? (3) the prediction problem: given a partial observation sequence and the radar model, how do we predict the next observation symbol, and compute the associated probability? and (4) the training problem: given only the observation sequence, how do we adjust (or train) the radar model so that it best describes how the observations came about?

Known solutions for the first three problems have been reviewed. The fourth problem is more difficult. Solving it would grant EW systems the capability to automatically reverse-engineer radar systems from intercepted radar waveforms.

The HMM concept is compatible with conventional receiver front-ends, and could eventually be inserted into a wide range of legacy ES, EA and ELINT systems. The only hardware requirement is the availability of a fast processor with sufficient memory.

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The growing proliferation and complexity of electromagnetic signals encountered in modern environments is greatly complicating the Electronic Support (ES), Electronic Attack (EA) and Electronic Intelligence (ELINT) functions. In conventional ES, EA and ELINT modeling, radar signals are expected to be stable or to correspond to radar modes. The concept examined in this report is the recognition of radar systems using hidden Markov modeling. The model is called hidden because the input symbols and state transitions of the radar system cannot be observed What is observed is the output of the finite state automaton. Hidden Markov models have been used extensively, and very successfully, for speech recognition. Four electronic warfare problems are examined: (1) the classification problem: given an observation signal and multiple competing radar models, how do we choose the model which best matches the observation signal? (2) the decoding problem: given the observation sequence and the radar model, how do we choose a state sequence which in some way best explains the observations? (3) the prediction problem. given a partial observation sequence and the radar model, how do we predict the next observation symbol, and compute the associated probability? and (4) the training problem: given only the observation sequence, how do we adjust (or train) the radar model so that it best describes how the observations came about? Known solutions for the first three problems are reviewed. The fourth problem is more difficult. Solving it would grant ES, EA and ELINT systems the capability to automatically reverse-engineer radar systems from intercepted radar waveforms

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Electronic Support
Electronic Attack
Electronic Intelligence
Electronic Warfare
Radar
Hidden Markov Model
State Machine
Finite State Automaton
Pattern Recognition

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